Time Dependence of the Decay Energy Spectrum for Stopped Cosmic-Ray Particles at Sea Level*

ELIHU BOLDT, PAUL STOLER, AND COSTA TSIPIS†
Rutgers, The State University, New Brunswick, New Jersey
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Within the indicated standard statistical uncertainties, the mean life (τ) for particles of the sea-level cosmic-ray muon beam, stopped in carbon, is independent of the decay energy classification (E in MeV).

$$\tau(29>E>11)/\tau(50>E>29)=1.01\pm0.03,$$
$$\tau(66>E>43)/\tau(E>26)=1.07\pm0.05,$$
$$\tau(E>26)=(2.109\pm0.032)\ \mu\text{sec}.$$

INTRODUCTION

THIS experiment consists of measurements that examine the temporal homogeneity of the decay process for the particles of the sea-level cosmic-ray muon beam, stopped in carbon. The *raison d'être* for this research is to be understood within the context of the history of experiments that established the characteristics of the weakly interacting component of the cosmic-ray flux.

The mass spectrum of charged cosmic-ray particles at sea level, excluding electrons, may be consistently attributed to particles of muon mass, with a small admixture of protons.[1] The ratio of positive to negative penetrating particles, with momentum greater than 1 GeV/c, is about[2] $1.2\rightarrow1.3$. The weakly interacting particles undergo beta decay, with a mean life of 2.22 ±0.02 μsec,[3] directly observed for positron emission. Hence, the gross picture that emerges is that the penetrating flux at sea level consists of muons, where the positive excess reflects the parent pion multiplicity at production, high in the atmosphere.[2]

In 1956, Alikhanian *et al.*[4] presented cosmic-ray evidence for the existence of a weakly interacting particle with a mass of 550 electron masses, at about a 0.5% abundance, relative to muons. This prompted a series of several extensive investigations of a possible fine structure to the composition of the cosmic-ray flux, particularly at sea level. As exhibited by some representative experiments[5-8] tabulated in Table I, these investigations set small upper limits to the abundance of an almost complete set of possible unusual objects in the cosmic-ray flux. However, these surveys did not eliminate the possible elusive presence of a particle with a mass comparable to that of the muon and which undergoes beta decay with a lifetime also comparable to that of the muon. Such a particle could be masked by the large background of muons. Yet, one of the first statistically precise determinations of the mean life of artificially produced positive muons yielded a number that was two standard deviations larger than the cosmic-ray value previously indicated. Although uncompelling statistically, this anomalous result, indicating a 2% discrepancy in mean life, prompted the investigation reported here.

TABLE I. Survey of experiments that define contamination limits for the cosmic-ray muon beam.

Characteristics of hypothetical contamination: Particle mass m (electron mass units)	Mean lifetime τ (sec)	Abundance upper limit, relative to muons
$\simeq550$	...	0.02%[a]
$100>m>30$	...	0.13%[b]
$900>m>400$	...	0.04%[b]
$m>60$	$10^{-2}>\tau>10^{-4}$	0.03%[c]
$m>300$	$10^{-3}>\tau>10^{-6}$	0.06%[d]
$m>210$ (with light meson decay product)	$10^{-3}>\tau>10^{-6}$	0.05%[d]

[a] See Ref. 5. [b] See Ref. 6. [c] See Ref. 7. [d] See Ref. 8.

If the positive muon lifetime were 2% longer than the observed lifetime for positive muon-like cosmic rays, then this would imply the presence of a shorter lived contamination among the cosmic-ray muons. For a hypothetical 10% contamination, the anomalous particle would be characterized by a mean life that is about 18% less than that for the muon. Hence, by separately examining the lifetime for events from different portions of the cosmic-ray muon beta decay

* Supported by a grant from the National Science Foundation.
† Present address: Columbia University, New York, New York.
[1] B. Rossi, *High-Energy Particles*, (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1952), p. 177.
[2] G. N. Fowler and A. W. Wolfendale, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1961), Vol. 46, Part 1, pp. 294–295.
[3] W. E. Bell and E. P. Hincks, Phys. Rev. **84**, 1243 (1951).
[4] A. I. Alikhanian, N. V. Shostakovich, A. T. Dadoin, V. N. Fedrov, and B. N. Deriagin, Zh. Eksperim. i Teor. Fiz. **31**, 955 (1956) [translation: Soviet Phys.—JETP **4**, 817 (1957)].
[5] I. B. McDiarmid, Phys. Rev. **115**, 1016 (1959).
[6] G. Fazio and M. Widgoff, Phys. Rev. **116**, 1263 (1959).
[7] G. Fazio and D. Ritson, Phys. Rev. **116**, 1267 (1959).
[8] E. Boldt, J. Hersil, Y. Pal, and J. Russell, Progr. Rept. MIT Lab. for Nucl. Sci., 1958, p. 54 (unpublished).
[9] R. A. Swanson, R. A. Lundy, V. L. Telegdi, and D. D. Yovanovitch, Phys. Rev. Letters **2**, 430 (1959). The value reported in this paper, 2.261 ± 0.007 μsec, was retracted in a postdeadline paper presented at the Washington, D. C. meeting of The American Physical Society, 1960.

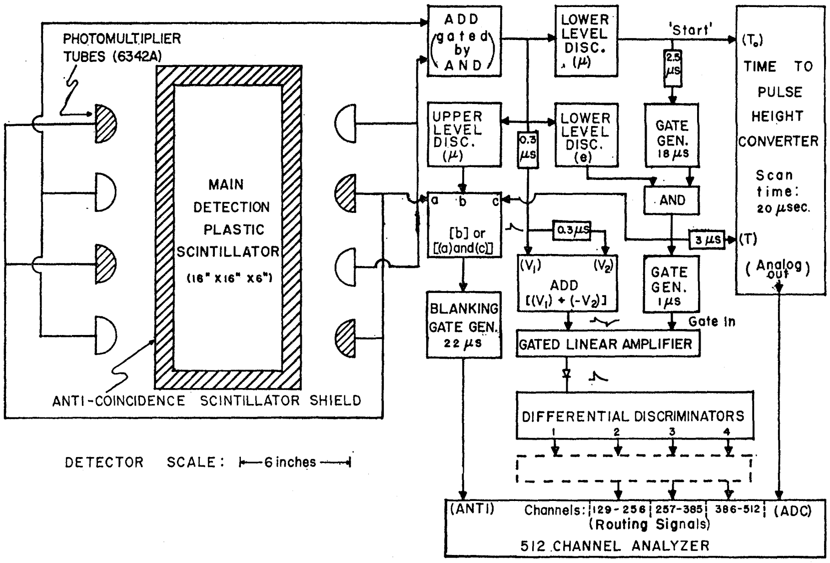

FIG. 1. A schematic representation of the detector configuration and the associated electronic circuit logic. The detector signal is delayed via 0.3-μsec transmission lines (RG 176/u) to allow for the indicated operations of delayed negative addition $[(V_1)+(-V_2)]$ and gated amplification. The timing pulses are delayed 2.5 and 3.0 μsec, via the action of univibrators, in order to ensure analysis on decay events only and that the "measure" operation occurs over the linear range of the time-to-pulse-height converter.

energy spectrum, it seemed feasible that such a contamination could be isolated. Since recent precise determinations[10] of the lifetime for artificially produced muons agree with the cosmic-ray value,[3] there no longer exists the aforementioned direct motivation for the presently reported investigation. However, by applying the indicated analysis to the decay events of weakly interacting cosmic-ray particles stopped in carbon there is exhibited here, for the first time, some direct experimental evidence to support the generally accepted conjecture that the homogeneity of time holds for weak interactions, in general,[11] and for the cosmic-ray muon beam, in particular.

For the homogeneity of time to be valid, it is necessary that the decay energy spectrum be independent of time, or equivalently, that the mean life of decay events be independent of the decay energy classification. Deviations from the homogeneity of time for the stopped particles of the cosmic-ray muon beam would ensue situations of the following type:

(1) Environmental aging of the bound negative muon as it traverses the nucleus of a mesic atom.
(2) Environmental aging of the muon in the presence of Coulomb fields.
(3) A mixture of two or more types of particles in the cosmic-ray muon beam (e.g., the muon from pion decay might be distinguishable from the muon coming from K-meson decay).
(4) Nonconservation of energy in the decay of the free muon; a variation of less than 4 MeV for the total energy of the final state could escape detection in experiments that measured the beta energy spectrum.[12,13]

The decay rate for the muon is proportional to the fifth power of the rest mass.[14] Therefore, mean-life measurements constitute a sensitive probe for anomalous muon mass values in the cosmic ray beam. For example, beta decay events characterized by a parent mass which is 10 MeV/c^2 more than that of the normal muon would exhibit a mean life that is 36% shorter than the standard value. Such possibilities were examined in these investigations by separately measuring the mean life for events exhibiting an energy release in the vicinity of the beta spectrum's end point.

EXPERIMENT

Unstable cosmic-ray particles were stopped in a 26-kg block of organic scintillation plastic (polyvinyltoluene). The scintillation light output arising from charged decay products that stop in this block constituted the decay energy signature of accepted events. This main detector was surrounded by a 4π anticoincidence shield for the rejection of escaped charged decay products. Light originating from the scintillation plastic sheets of the shield was distinguished from the light output of the main scintillator by means of optical filters, in a manner previously described.[15] An end view of this configuration of scintillators is indicated in Fig. 1.

The valid measurement of a temporal inhomogeneity in a physical process is ensured when the measurement process itself is homogeneous in time. For this experiment each event is characterized by: (1) the energy

[10] R. Lundy, Phys. Rev. **125**, 1686 (1962). Reported value of τ for μ^+ is (2.203±0.004) μsec. The weighted mean of five recent determinations is cited as (2.210±0.002) μsec.

[11] W. Pauli, *Structure et Propriétés des Noyaux Atomiques* (Institut International de Physique Solvay-7, Brussels, 1933), p. 324.

[12] S. Weinberg, Phys. Rev. **128**, 1457 (1962).

[13] W. F. Dudziak, R. Sagane, and J. Vedder, Phys. Rev. **114**, 336 (1959).

[14] T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 593 (1957).

[15] E. Boldt and C. Tsipis, Rev. Sci. Instr. **32**, 280 (1961).

dissipated by the parent particle in stopping; (2) the energy released via the charged decay products, and (3) the time interval between the instants of stopping and decay. Therefore, the validity of this experiment requires (1) the energy scale to be independent of time, and (2) the time base to be independent of the energy classification.

A schematic representation of the experiment is exhibited in Fig. 1. The time interval between the instants of stopping and decay is linearly converted to a voltage by a time-to-pulse-height converter.[16] This analog voltage is entered into the ADC input of a 512-channel pulse-height analyzer.[17] The resultant digital information is subsequently stored in a quadrant of the analyzer memory upon a storage command signal. Each one of the last three quadrants of the memory is commanded by a separate routing signal representing a particular energy classification for the decay event. When none of the prescribed energy classifications is pertinent, the digital information is automatically stored in the first quadrant of the memory. Since the measurement of decay energy is utilized solely in connection with routing the storage of information that is already digitized, the effective time base for each event is independent of the energy classification.

The input to the ADC of the multichannel analyzer is rejected for analysis by the presence of a blanking gate on the anti-input. This blanking gate is initiated either by (1) a decay event signal from the anticoincidence shield, indicating a traversal of the scintillator boundary by a charged particle, or by (2) a signal from the upper level discriminator (μ), indicating an excessive amount of energy associated with the stopping parent particle.[18] A summary of operating conditions is presented in Tables II and III.

The energy of the decay event is classified by four associated differential discriminators, designated Channel 1, Channel 2, Channel 3, and Channel 4. The reference levels of these discriminators are fixed by mercury cells to ensure long-time stability and the discrimination edges, under operating conditions, exhibit a sharpness corresponding to approximately 0.5 MeV. For a particular run, the output signals from three of these discriminators constitute the memory storage commands for the analyzer. The discrimination levels are set with regard to the end point energy (53 MeV) of the muon beta-decay spectrum. An energy classification well beyond this end point, Channel 4, and an energy classification well within the normal beta spectrum, Channel 2, are included in each run. The linear amplifier preceeding these discriminators is gated only when a decay event is indicated. Since these devices that perform the decay energy classification are not exposed to any signals associated with the stopping parent particle, the instrumental energy scale is necessarily independent of the time elapsed between the stopping of the primary particle and the subsequent decay.

A signal from the main detector is accepted, under the "ADD" operation, only when there are signals simultaneously present, meeting the "AND" requirement, on two independent sets of photomultiplier tubes. This eliminates spurious decay events arising from after-pulses in the photomultiplier tubes. However, this does not eliminate the occurrence of an after-pulse superposed upon a genuine decay-event signal. When this situation occurs, the energy reading of the decay event is shifted by the amplitude of the after-pulse. However, an after-pulse attendant to a stopping particle signal is incoherent with respect to a decay event signal. Hence, if each signal is subtracted in turn, after a short delay (0.3 μsec), then the after-pulses will destructively interfere with the genuine decay signal as often as they interfere constructively, and the average energy shift will vanish. The after-pulses are thereby reduced to noise. As indicated in Fig. 1, the operation $[(V_1)+(-V_2)]$ is that of delayed negative addition, where (V_2) is (V_1) delayed by 0.3 μsec.

The temporal homogeneity of the energy scale, over the operating time base interval, is monitored by the distribution of observed events associated with Channel 4. For both Run A and Run B, this channel represents events well beyond the muon beta end point energy, and all the observed counts can be consistently attributed to the random association in time of uncor-

TABLE II. Operating conditions and results for Run A (with anticoincidence shield).

Energy channel	1	2	4
Energy interval (MeV)	$11 \rightarrow 29$	$29 \rightarrow 50$	$79 \rightarrow 114$
Portion of theoretical muon[a] beta spectrum ($\rho = \frac{3}{4}$)	23%	63%	0
Shield rejection of delayed events	27%	37%	82%
Accepted Delayed Events: ($2.6 \rightarrow 9.4$) μsec 20 time units	3819	7519	78
Accepted Delayed Events: ($9.4 \rightarrow 16.2$) μsec 20 time units	308	492	85
Background expectation (20 time units)	168	219	82
τ (μsec)	2.09	2.07	...
Energy resolution (standard-deviation/mean) at calibration (87 MeV)			12%
Detected energy released by parent particle in stopping (MeV)			35—114
Unit time interval (μsec)			0.34
Running clock time (h)			193.7
τ (Channel 1)/τ(Channel 2)			1.01 ± 0.03[b]

[a] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[b] Statistical standard deviation.

[16] J. Fischer and A. Lundby, Rev. Sci. Instr. **31**, 10 (1960).

[17] Nuclear Data 512-channel analyzer, model N. D. 120, on loan from Bell Telephone Laboratory, Murray Hill, New Jersey.

[18] The upper level discriminator μ was set sufficiently low to eliminate most stopping particles associated with cosmic-ray showers.

TABLE III. Operating conditions and results for Run B (without anticoincidence shield).

Energy channel	2	3	4
Energy interval (MeV)	$26 \rightarrow 39$	$43 \rightarrow 65$	$66 \rightarrow 88$
Portion of theoretical muon[a] beta spectrum ($\rho = \frac{3}{4}$)	···	37%	0
Accepted Delayed Events: ($2.6 \rightarrow 9.4$) μsec, 21 time units	10 727	2471	317
Accepted Delayed Events: ($9.4 \rightarrow 16.2$) μsec, 21 time units	730	569	300
Background Expectation (21 time units): Internal	308	470	···
Background Expectation (21 time units): External	346 ± 14[b]	498 ± 21[b]	···
τ (μsec)	2.109 ± 0.032[b]	2.25 ± 0.11[b]	···
Total observed delayed events (2.6–16.2 μsec) with energy detected in interval $16 \rightarrow 65$ MeV			32444
Energy resolution (standard deviation/mean) at calibration (87 MeV)			12%
Detected energy released by parent particle in stopping (MeV)			22–88
Unit time interval (μsec)			0.3221
Running clock time (h)			211
τ(Channel 3)/τ(Channel 2)			1.07 ± 0.05[b]

[a] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[b] Statistical standard deviation.

related cosmic-ray particles traversing the detector. For Run A, the anticoincidence shield eliminates most of such events since the shield performs at an over-all efficiency of 82%. However, Run B was carried out with a relaxation of the anticoincidence requirement in order to gain a higher counting rate for both Channel 3 and Channel 4, and to exhibit, with reasonable statistical significance, the random distribution in time for events of the latter energy category. As shown in Table IV, the chi-square test for randomness was satisfactorily met for these data.

TABLE IV. Time scale for Run B.

	Measurement performed		
	Before run (time calibration)	During run (Channel 4)	After run (time calibration)
Total number of recorded counts in 42 unit time intervals	60229	617	195901
χ^2 for observed distribution among the 42 unit time intervals	56.9	36.3	44.6
Expected value of χ^2 for a random distribution	41 ± 9[a]	41 ± 9[a]	41 ± 9[a]
Effective live-time per unit time interval (μsec)	0.3204	···	0.3237
Mean effective live-time per unit time interval during run (μsec)	···	0.3221	···

[a] Statistical standard deviation.

The time base for Run B was calibrated by initiating the time-to-pulse-height converter on cosmic-ray particles, as usual, and sampling the time of pulses derived from an independent freely running pulser. The pulser was monitored by a crystal controlled electronic counter. Results of this calibration and tests for linearity and randomness are summarized in Table IV. A unit time interval corresponds to a single channel of the 512-channel analyzer.[17]

The energy calibration was effected by recording the pulse-height distribution of minimum ionizing cosmic-ray particles restricted to a well-defined trajectory through the detector by means of a telescope. The change in response over extreme regions of the scintillator block was less then the resolution. The linearity was checked by noting, within the resolution, the proper end point for the beta decay spectrum of the muon.

DISCUSSION

The parent particles are stopped in a scintillation medium that is entirely carbon and hydrogen. As far as negative muons are concerned, only carbon is effective.[19] When a negative muon stops in the vicinity of a hydrogen nucleus the two particles form a neutral system[20]; within the lifetime of the muon, this system makes several thousand collisions with carbon nuclei in which the muon may attach itself to the carbon. A negative muon takes about 10^{-10} seconds to slow down, be captured, and cascade into the K orbit of a carbon atom.[19] While in the K orbit of carbon, the relative probability of decay to that of nuclear capture is about 10/1.

A positive muon that stops in the scintillator probably shares an electron with the surrounding atoms in a covalent-type bond; positive muons stopped in carbon do not form much muonium.[21] The mean life

[19] G. T. Reynolds, D. B. Scarl, R. A. Swanson, J. R. Waters, and R. A. Zdanis, Phys. Rev. **129**, 1790 (1963).
[20] H. K. Ticho, Phys. Rev. **74**, 1337 (1948).
[21] J. Rainwater, Ann. Rev. Nucl. Sci. **7**, 7 (1957).

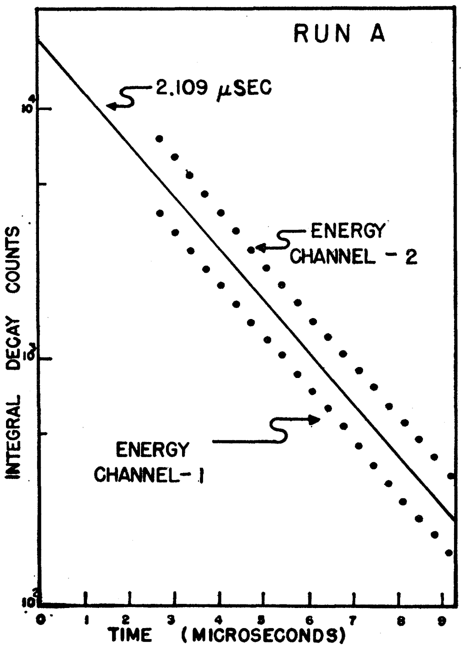

FIG. 2. Integral decay plots of data from Run A, over 20 unit time intervals, corrected for an expected background rate (counts per unit time interval) of 8.4 for Channel 1 and 11.0 for Channel 2. A unit time interval is 0.34 μsec. The solid line indicates a mean life $\tau=2.109$ μsec.

and decay energy spectrum for these muons are expected to be essentially the same as for free positive muons.[22]

With a statistical precision of about 0.2%, Eckhause *et al.*[23] measured the mean life of negative muons in carbon to be 2.026 μsec as compared with 2.202 μsec for the positive muons. For a 20% positive excess[2] in the cosmic-ray muon beam at sea level, the mean lifetime of muons stopped in carbon, referred to the zero-time of stopping, would therefore be 2.123 μsec. Since the resulting decay law represents a superposition of two exponential decay laws, the decay probability for the mixture will be a slowly varying function of time. For times comparable to the mean life, the decay will appear essentially exponential, with a characteristic mean life (τ) of 2.123 μsec.

The beta-decay energy spectrum of negative muons bound to carbon is expected to be the same as that for free muons.[24] Hence, the mean life of the cosmic-ray mixture of muons stopped in carbon is expected to be independent of the energy classification for the decay process. Therefore, this experiment is restricted to the investigation of that class of temporal inhomogeneities associated with a decay energy spectrum that changes in time. The contamination of a particle with a decay energy spectrum and mean life differing from those of the normal muon would yield the requisite inhomogeneity.

RESULTS

The results of this experiment are summarized in Tables II and III. The indicated mean lifetimes τ were obtained by assuming a differential distribution in time t of the form $(ae^{-t/\tau}+b)$ and analyzing the data according to a procedure similar to that prescribed by Peierls[25]; see Appendix. This analysis also yielded an estimate for the background rate b, designated "internal." The continuous rate of uncorrelated cosmic-ray events in each energy category was independently monitored for Run B and, in association with the data of Channel 4, gives an additional estimate of the background rates, designated "external", for all other channels.

Run A compared directly the mean lifetimes for two major portions of the muon beta decay energy spectrum; see Table II. For this run, Channel 1 classified decay events in the energy interval 11–29 MeV and Channel 2 classified those in the interval 29–50 MeV. The result of this measurement is: $\tau_1/\tau_2=1.01\pm0.03$.

Run B compared the mean lifetime for events in the vicinity of the muon beta end-point energy, Channel 3, with the mean lifetime for events well within the main body of the beta spectrum, Channel 2. The result of this measurement is: $\tau_3/\tau_2=1.07\pm0.05$.

As pointed out by Kuščer *et al.*,[26] an integral plot of the decay distribution presents a sensitive visual representation of the mean life τ appropriate to the data. Such integral decay curves for Channel 1 and Channel 2 of Run A are presented together in Fig. 2. Corresponding curves for Channel 2 and Channel 3 of Run B are presented together in Fig. 3.

It is important to note that the anticoincidence shield was not used for Run B. Therefore, the upper limit for the energy classification of Channel 2 did not correspond to the upper limit of the decay energy of all events accepted into this category. However, setting this upper limit for Channel 2 below the beta end-point energy had the desirable effect of reducing the background contamination for this channel. Since the beta end-point energy of the muon is approximately midway in Channel 3, the energy classification for this channel is valid for muons, in spite of the lack of an anticoincidence rejection of escaping particles. The unavoidable resultant large background in this region of relatively few observed decay events, did substantially contribute to the indicated statistical uncertainty of the result. As exhibited in Table V, the results of Run B are satisfactorily stationary with respect to a variation in the portion of the data analyzed.

This experiment's best estimate of the mean life τ

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[22] R. M. Tennent, *Progress in Elementary Particle and Cosmic Ray Physics* (North-Holland Publishing Company, Amsterdam, 1960), Vol. 5, p. 370.

[23] M. Eckhause, T. A. Filippas, R. B. Sutton, and R. E. Welsh, Bull. Am. Phys. Soc. **7**, 489 (1962).

[24] H. Überall, Phys. Rev. **119**, 365 (1960).

[25] R. Peierls, Proc. Roy. Soc. (London) **A149**, 467 (1935).

[26] I. Kuščer, M. V. Milhailovic, and E. C. Pask, Phil. Mag. **2**, 998 (1957).

TABLE V. Values of τ abstracted from various portions of the data of Run B.

Analysis time interval: $t \to (t+\Delta t)$		τ (μsec)	
t (μsec)	Δt (μsec)	Channel 2	Channel 3
2.6	13.5	2.109	2.25
2.9	11.6	2.103	2.18
3.2	11.6	2.094	2.21
3.6	11.6	2.099	2.20
3.2	12.2	2.081	2.23
3.2	11.0	2.100	2.18
3.2	10.3	2.115	2.19
3.2	9.7	2.113	2.23

for the cosmic-ray mixture of muons stopped in carbon is obtained from the data of Channel 2, Run B. The previously indicated anticipated value, 2.123 μsec, is consistent with the mean life measured in this experiment, viz; 2.109±0.032 μsec. The differential distributions in time of all counts observed for Channel 2 and Channel 4, Run B, are presented in Fig. 4.

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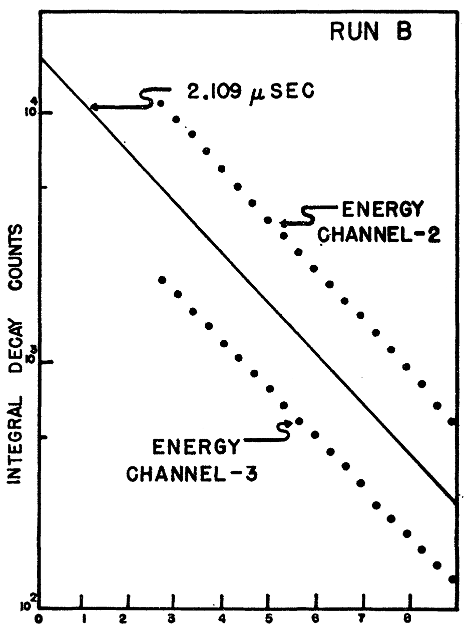

FIG. 3. Integral decay plots of data from Run B, over 21 unit time intervals, corrected for an expected background rate (counts per unit time interval) of 14.7 for Channel 2 and 22.4 for Channel 3. A unit time interval is 0.3221 μsec. The solid line indicates a mean life $\tau = 2.109$ μsec.

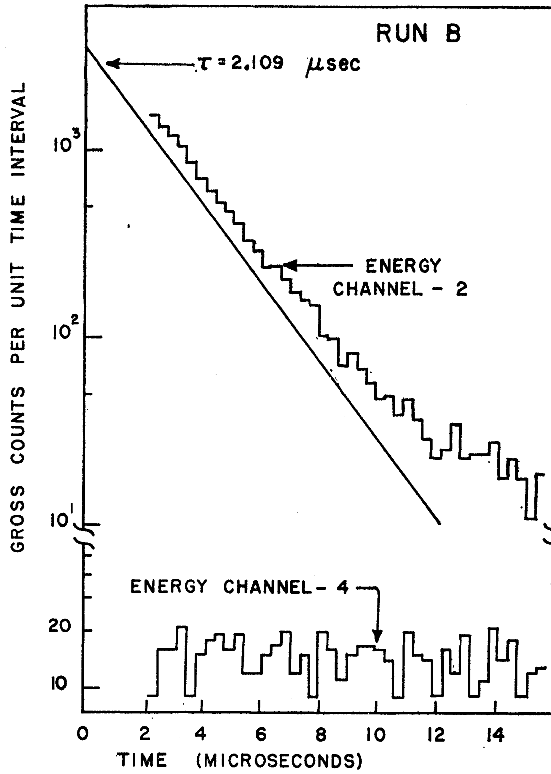

FIG. 4. Total observed counts per unit time interval, over 42 unit time intervals, for Channel 2 and Channel 4 of Run B. A unit time interval is 0.3221 μsec.

APPENDIX A

Consider events observed over a time interval $2T$, resulting from an exponential decay process, characterized by a mean life τ, plus a random background. The expectation value $\langle t_d \rangle$ of the lifetime of decay events, observed over a time T, is related to the characteristic mean life (τ) as follows.[25]

$$\tau - \langle t_d \rangle = T/(e^{T/\tau} - 1). \tag{A1}$$

Pertinent measured quantities are:

$\langle t_1 \rangle \equiv$ The mean life of *all* events observed during the first time interval T.

$\langle t_2 \rangle \equiv$ The mean life of *all* events observed during the subsequent interval T.

$N_1 \equiv$ The total number of events observed during the first time interval T.

$N_2 \equiv$ The total number of events observed during the subsequent interval T.

The desired expectation value $\langle t_d \rangle$ may be obtained from these measured quantities.

$$\langle t_d\rangle(N_1-N_2)=N_1\langle t_1\rangle-N_2\langle t_2\rangle. \quad \text{(A2)}$$

The mean life τ, characteristic of the decay process, is then obtained as a solution of Eq. (A1).

Let δ be the standard statistical uncertainty on $\langle t_d\rangle$. This may be expressed directly in terms of measured quantities, viz;

$$\delta^2(N_1-N_2)^2 =N_1[\langle(t_1)^2\rangle-(\langle t_1\rangle)^2+(\langle t_1\rangle-\langle t_d\rangle)^2] +N_2[\langle(t_2)^2\rangle-(\langle t_2\rangle)^2+(\langle t_2\rangle-\langle t_d\rangle)^2], \quad \text{(A3)}$$

where $\langle(t_i)^2\rangle\equiv$ the mean of the squared lifetime of *all* events observed during the ith interval T.

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Regge Poles and the Photoproduction of Pions*

R. W. CHILDERS AND W. G. HOLLADAY
Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee
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The photoproduction of pions, $\gamma+N\to\pi+N$, is examined from the standpoint of the Regge pole hypothesis and the Mandelstam representation. The asymptotic behavior of the forward scattering amplitudes is determined in terms of the Regge trajectories of the $\gamma+\pi\to N+\bar{N}$ channel. The ρ, ϕ, and ω trajectories are included in the description of the photoproduction of neutral pions, whereas only the ρ and π trajectories contribute to the photoproduction of charged pions. In the case of backward scattering, asymptotic representations of the scattering amplitudes are controlled by the trajectories of the $\gamma+N\to\pi+N$ channel by crossing. Finally, generalized Pomeranchuk relations are established for the differential cross sections in the forward and backward directions for the various charge configurations of the photoproduction channel. In particular, we have the following interesting results: (1) The differential cross sections for $\gamma+p\to n+\pi^+$ and $\gamma+n\to p+\pi^-$ are asymptotically equal in the forward and backward directions; (2) the differential cross sections for $\gamma+p\to p+\pi^0$ and $\gamma+n\to n+\pi^0$ are asymptotically equal in the backward direction.

I. INTRODUCTION

WITH the exception of selection rules, the underlying physical principles of elementary-particle physics are currently being expressed in terms of analyticity properties of transition amplitudes in a manner consistent with unitarity. In fact, for strongly interacting systems, the principle of maximal analyticity in linear momentum has been frequently invoked.[1] The resulting description, i.e., the Mandelstam representation and unitarity, is incomplete at least to the extent that the behavior of the scattering amplitude at infinity remains undetermined. Complex angular momentum may be useful in this respect since Regge[2] has shown that the meromorphicity and asymptotic boundedness of the partial-wave amplitudes continued to complex J provide boundary conditions for the scattering amplitude at infinity. Although Regge's work is for potential scattering and relativistic proofs of certain aspects of this program are still lacking, it is desirable, nevertheless, to investigate the consequences of this approach since the equations obtained for the scattering amplitudes are simple in form and are subject to experimental verification.

These and other considerations have led us to examine a process in which both strong and electromagnetic interactions enter, namely the photoproduction of pions. The basic assumption is that the particles mediating the strong interaction correspond to certain Regge trajectories in the complex J plane.

Our main purpose is to demonstrate the existence of generalized Pomeranchuk relations for the photoproduction of pions. These relations do not necessarily pertain to total cross sections of particle and antiparticle reactions, as did the original Pomeranchuk theorems. Instead, it is recognized that the fundamental mechanism responsible for the Pomeranchuk theorems, namely a dominant Regge trajectory, may also be the author of other asymptotic symmetries. Wagner and Sharp,[3,4] for instance, have discussed such asymptotic relationships between the differential cross sections for the direct and crossed channels of several reactions. For photoproduction the equality of the differential cross sections for the direct and crossed channels is guaranteed at all energies by invariance under charge conjugation. We, therefore, turn to the particular charge configurations present in a given channel, and it is found that they satisfy asymptotic symmetries of this type.

The general plan of the paper is as follows. In Sec. II, the basic kinematics are outlined and the amplitudes

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